Expression des opérateurs différentiels

Soient f, champ scalaire, et $\overrightarrow{\mathrm{A}}$, champ vectoriel, fonctions des trois coordonnées spatiales. On note V le champ scalaire ne dépendant que de r (des cylindriques ou des sphériques).

1 En coordonnées cartésiennes

$$\overrightarrow{\mathrm{grad}}\ f = \frac{\partial f}{\partial x}\,\overrightarrow{u_x} + \frac{\partial f}{\partial y}\,\overrightarrow{u_y} + \frac{\partial f}{\partial z}\,\overrightarrow{u_z}$$

$$\mathrm{div}\ \overrightarrow{\mathrm{A}} = \frac{\partial \mathrm{A}_x}{\partial x} + \frac{\partial \mathrm{A}_y}{\partial y} + \frac{\partial \mathrm{A}_z}{\partial z}$$

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\overrightarrow{\mathrm{rot}}\ \overrightarrow{\mathrm{A}} = \left[\frac{\partial \mathrm{A}_z}{\partial y} - \frac{\partial \mathrm{A}_y}{\partial z}\right]\overrightarrow{u_x} + \left[\frac{\partial \mathrm{A}_x}{\partial z} - \frac{\partial \mathrm{A}_z}{\partial x}\right]\overrightarrow{u_y} + \left[\frac{\partial \mathrm{A}_y}{\partial x} - \frac{\partial \mathrm{A}_x}{\partial y}\right]\overrightarrow{u_z}$$

2 En coordonnées cylindriques

$$\overrightarrow{\mathrm{grad}}\ f = \frac{\partial f}{\partial r}\,\overrightarrow{u_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\overrightarrow{u_\theta} + \frac{\partial f}{\partial z}\,\overrightarrow{u_z}$$

$$\mathrm{div}\ \overrightarrow{\mathrm{A}} = \frac{1}{r}\frac{\partial (r\mathrm{A}_r)}{\partial r} + \frac{1}{r}\frac{\partial \mathrm{A}_\theta}{\partial \theta} + \frac{\partial \mathrm{A}_z}{\partial z}$$

$$\Delta \mathrm{V} = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}f}{\mathrm{d}r}\right) = \frac{\mathrm{d}^2 f}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}f}{\mathrm{d}r}$$

$$\overrightarrow{\mathrm{rot}}\ \overrightarrow{\mathrm{A}} = \frac{1}{r}\left[\frac{\partial \mathrm{A}_z}{\partial \theta} - \frac{\partial (r\mathrm{A}_\theta)}{\partial z}\right]\overrightarrow{u_r} + \left[\frac{\partial \mathrm{A}_r}{\partial z} - \frac{\partial \mathrm{A}_z}{\partial r}\right]\overrightarrow{u_\theta} + \frac{1}{r}\left[\frac{\partial (r\mathrm{A}_\theta)}{\partial r} - \frac{\partial \mathrm{A}_r}{\partial \theta}\right]\overrightarrow{u_z}$$

3 En coordonnées sphériques

$$\overrightarrow{\mathrm{grad}}\ f = \frac{\partial f}{\partial r}\,\overrightarrow{u_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\overrightarrow{u_\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\,\overrightarrow{u_\varphi}$$

$$\mathrm{div}\ \overrightarrow{\mathrm{A}} = \frac{1}{r^2}\frac{\partial (r^2\mathrm{A}_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial \mathrm{A}_\theta \sin\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \mathrm{A}_\varphi}{\partial \varphi}$$

$$\Delta \mathrm{V} = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}f}{\mathrm{d}r}\right) = \frac{\mathrm{d}^2 f}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}f}{\mathrm{d}r}$$

$$\overrightarrow{\mathrm{rot}}\ \overrightarrow{\mathrm{A}} = \begin{cases} \dfrac{1}{r^2\sin\theta}\left[\dfrac{\partial (r\mathrm{A}_\varphi \sin\theta)}{\partial \theta} - \dfrac{\partial (r\mathrm{A}_\theta)}{\partial \varphi}\right] & \text{selon } \overrightarrow{u_r} \\ \dfrac{1}{r\sin\theta}\left[\dfrac{\partial \mathrm{A}_r}{\partial \varphi} - \dfrac{\partial (r\mathrm{A}_\varphi \sin\theta)}{\partial r}\right] & \text{selon } \overrightarrow{u_\theta} \\ \dfrac{1}{r}\left[\dfrac{\partial (r\mathrm{A}_\theta)}{\partial r} - \dfrac{\partial \mathrm{A}_r}{\partial \theta}\right] & \text{selon } \overrightarrow{u_\varphi} \end{cases}$$